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APPLICATION OF KAMAL TRANSFORM FOR SOLVING POPULATION GROWTH AND DECAY PROBLEMS

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ABSTRACT
The population growth and decay problems arise in the field of chemistry, physics, biology, social science, zoology etc. In this paper, we used Kamal transform for solving population growth and decay problems and some applications are given in order to demonstrate the effectiveness of Kamal transform for solving population growth and decay problems.

Keywords: Kamal transform, Inverse Kamal transform, Population growth problem, Decay problem, Half-life.

I.   INTRODUCTION

The population growth (growth of a plant, or a cell, or an organ, or a species) is governed by the first order linear ordinary differential equation [1-10]
$$\frac{dN}{dt} = kN \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$
with initial condition as
$$N(t_0) = N_0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

where k is a positive real number, N is the amount of population at time t and N_0 is the initial population at time t_0. Equation (1) is known as the Malthusian law of population growth.

Mathematically the decay problem of the substance is defined by the first order linear ordinary differential equation [7, 9-10]
$$\frac{dN}{dt} = -kN \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$
with initial condition as
$$N(t_0) = N_0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4)$$

where N is the amount of substance at time t, k is a positive real number and N_0 is the initial amount of the substance at time t_0.

In equation (3), the negative sign in the R.H.S. is taken because the mass of the substance is decreasing with time and so the derivative $\frac{dN}{dt}$ must be negative.

The Kamal transform of the function $F(t)$ is defined as [11]:

$$K\{F(t)\} = \int_0^\infty F(t)e^{\frac{-t}{v}} dt = G(v), t \geq 0, k_1 \leq v \leq k_2$$

where K is Kamal transform operator.

The Kamal transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Kamal transform of the function $F(t)$.

Abdelilah and Hassan [12] applied Kamal transform for solving partial differential equations. Fadhil [13] discussed the convolution for Kamal and Mahgoub transforms. Taha et. al. [14] defined the dualities between Kamal & Mahgoub integral transforms and some famous integral transforms. Aggarwal et al. [15] discussed a new application of Kamal transform for solving linear Volterra integral equations. Gupta et al. [16] gave the solution of linear partial integro-differential equations using Kamal transform. Aggarwal [17] defined Kamal transform of Bessel's functions. Application of Kamal transform for solving linear Volterra integral \equations of first kind was given by Aggarwal et al. [18].

The aim of this work is to finding the solution of population growth and decay problems using Kamal transform without large computational work.

II. LINEARITY PROPERTY OF KAMAL TRANSFORM [17-18]

If $K\{F(t)\} = H(v)$ and $K\{G(t)\} = I(v)$ then $K\{aF(t) + bG(t)\} = aK\{F(t)\} + bK\{G(t)\}$
$\Rightarrow K\{aF(t) + bG(t)\} = aH(v) + bI(v),$

where a, b are arbitrary constants.

III. KAMAL TRANSFORM OF SOME ELEMENTARY FUNCTIONS [15-18]

S.N.	$F(t)$	$K\{F(t)\} = G(v)$
1.	1	v
2.	t	v^2
3.	t^2	$2!\, v^3$
4.	$t^n, n\epsilon N$	$n!\, v^{n+1}$
5.	$t^n, n > -1$	$\Gamma(n+1)v^{n+1}$
6.	e^{at}	$\dfrac{v}{1-av}$
7.	$sinat$	$\dfrac{av^2}{1+a^2v^2}$
8.	$cosat$	$\dfrac{v}{1+a^2v^2}$
9.	$sinhat$	$\dfrac{av^2}{1-a^2v^2}$
10.	$coshat$	$\dfrac{v}{1-a^2v^2}$

IV.   INVERSE KAMAL TRANSFORM [15-18]

If $K\{F(t)\} = G(v)$ then $F(t)$ is called the inverse Kamal transform of $G(v)$ and mathematically it is defined as $F(t) = K^{-1}\{G(v)\}$, where K^{-1} is the inverse Kamal transform operator.

V.   LINEARITY PROPERTY OF INVERSE KAMAL TRANSFORM

If $K^{-1}\{H(v)\} = F(t)$ and $K^{-1}\{I(v)\} = G(t)$ then
$K^{-1}\{aH(v) + bI(v)\} = aK^{-1}\{H(v)\} + bK^{-1}\{I(v)\}$
$\Rightarrow K^{-1}\{aH(v) + bI(v)\} = aF(t) + bG(t)$, where a, b are arbitrary constants.

VI.   INVERSE KAMAL TRANSFORM OF SOME ELEMENTARY FUNCTIONS [15-18]

S.N.	$G(v)$	$F(t) = K^{-1}\{G(v)\}$
1.	v	1
2.	v^2	t
3.	v^3	$\dfrac{t^2}{2!}$
4.	$v^{n+1}, n\epsilon N$	$\dfrac{t^n}{n!}$
5.	$v^{n+1}, n > -1$	$\dfrac{t^n}{\Gamma(n+1)}$
6.	$\dfrac{v}{1-av}$	e^{at}
7.	$\dfrac{v^2}{1+a^2v^2}$	$\dfrac{sinat}{a}$
8.	$\dfrac{v}{1+a^2v^2}$	$cosat$
9.	$\dfrac{v^2}{1-a^2v^2}$	$\dfrac{sinhat}{a}$
10.	$\dfrac{v}{1-a^2v^2}$	$coshat$

VII.   KAMAL TRANSFORM OF THE DERIVATIVES OF THE FUNCTION $F(t)$ [11]

If $K\{F(t)\} = G(v)$ then
   a) $K\{F'(t)\} = \dfrac{1}{v}G(v) - F(0)$
   b) $K\{F''(t)\} = \dfrac{1}{v^2}G(v) - \dfrac{1}{v}F(0) - F'(0)$
   c) $K\{F^{(n)}(t)\} = \dfrac{1}{v^n}G(v) - \dfrac{1}{v^{n-1}}F(0) - \dfrac{1}{v^{n-2}}F'(0) \dots \dots - F^{(n-1)}(0)$

VIII.   KAMAL TRANSFORM FOR POPULATION GROWTH PROBLEM

In this section, we present Kamal transform for population growth problem given by (1) and (2).
Applying the Kamal transform on both sides of (1), we have
$K\left\{\dfrac{dN}{dt}\right\} = kK\{N(t)\} \dots \dots \dots \dots \dots \dots \dots \dots \dots . (5)$

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Now applying the property, Kamal transform of derivative of function, on (5), we have

$$\frac{1}{v}K\{N(t)\} - N(0) = kK\{N(t)\} \dots \dots \dots \dots \dots (6)$$

Using (2) in (6) and on simplification, we have

$$\left(\frac{1}{v} - k\right)K\{N(t)\} = N_0$$

$$\Rightarrow K\{N(t)\} = \frac{vN_0}{(1 - kv)} \dots \dots \dots \dots \dots \dots \dots . (7)$$

Operating inverse Kamal transform on both sides of (7), we have

$$N(t) = N_0 K^{-1}\left\{\frac{v}{(1 - kv)}\right\} = N_0 e^{kt} \dots \dots \dots (8)$$

which is the required amount of the population at time t.

IX.   KAMAL TRANSFORM FOR DECAY PROBLEM

In this section, we present Kamal transform for decay problem which is mathematically given by (3) and (4).
Applying the Kamal transform on both sides of (3), we have

$$K\left\{\frac{dN}{dt}\right\} = -kK\{N(t)\} \dots \dots \dots \dots \dots \dots \dots . \dots . (9)$$

Now applying the property, Kamal transform of derivative of function, on (9), we have

$$\frac{1}{v}K\{N(t)\} - N(0) = -kK\{N(t)\} \dots \dots \dots \dots (10)$$

Using (4) in (10) and on simplification, we have

$$\left(\frac{1}{v} + k\right)K\{N(t)\} = N_0$$

$$\Rightarrow K\{N(t)\} = \frac{vN_0}{(1 + kv)} \dots \dots \dots \dots \dots \dots . \dots . (11)$$

Operating inverse Kamal transform on both sides of (11), we have

$$N(t) = N_0 K^{-1}\left\{\frac{v}{(1 + kv)}\right\}$$

$$\Rightarrow N(t) = N_0 e^{-kt} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (12)$$

which is the required amount of substance at time t.

X.   APPLICATIONS

In this section, some applications are given in order to demonstrate the effectiveness of Kamal transform for solving
population growth and decay problems.

Application:1 The population of a city grows at a rate proportional to the number of people presently living in the
city. If after two years, the population has doubled, and after three years the population is 20,000, estimate the number
of people initially living in the city.

This problem can be written in mathematical form as:

$$\frac{dN(t)}{dt} = kN(t) \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots . (13)$$

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where N denote the number of people living in the city at any time t and k is the constant of proportionality. Consider N_0 is the number of people initially living in the city at $t = 0$.

Applying the Kamal transform on both sides of (13), we have
$$K\left\{\frac{dN(t)}{dt}\right\} = kK\{N(t)\} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \ldots . (14)$$

Now applying the property, Kamal transform of derivative of function, on (14), we have
$$\frac{1}{v}K\{N(t)\} - N(0) = kK\{N(t)\} \ldots \ldots \ldots \ldots \ldots . . (15)$$

Since at $t = 0, N = N_0$, so using this in (15), we have
$$\frac{1}{v}K\{N(t)\} - N_0 = kK\{N(t)\}$$
$$\Rightarrow \left(\frac{1}{v} - k\right) K\{N(t)\} = N_0$$
$$\Rightarrow K\{N(t)\} = \frac{vN_0}{(1 - kv)} \ldots \ldots \ldots \ldots \ldots \ldots \ldots . (16)$$

Operating inverse Kamal transform on both sides of (16), we have
$$N(t) = N_0 K^{-1}\left\{\frac{v}{(1 - kv)}\right\} = N_0 e^{kt} \ldots \ldots \ldots \ldots . (17)$$

Now at $= 2, N = 2N_0$, so using this in (17), we have
$$2N_0 = N_0 e^{2k}$$
$$\Rightarrow e^{2k} = 2$$
$$\Rightarrow k = \frac{1}{2} log_e 2 = 0.347 \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . \ldots (18)$$

Now using the condition at $= 3, N = 20,000$, in (17), we have
$$20,000 = N_0 e^{3k} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . (19)$$

Putting the value of k from (18) in (19), we have
$$20,000 = N_0 e^{3 \times 0.347}$$
$$\Rightarrow 20,000 = 2.832 N_0$$
$$\Rightarrow N_0 \simeq 7062 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (20)$$

which are the required number of people initially living in the city.

Application:2 A radioactive substance is known to decay at a rate proportional to the amount present. If initially there is 100 milligrams of the radioactive substance present and after two hours it is observed that the radioactive substance has lost 10 percent of its original mass, find the half life of the radioactive substance.

This problem can be written in mathematical form as:
$$\frac{dN(t)}{dt} = -kN(t) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . \ldots . \ldots . (21)$$

where N denote the amount of radioactive substance at time t and k is the constant of proportionality. Consider N_0 is the initial amount of the radioactive substance at time $t = 0$.
Applying the Kamal transform on both sides of (21), we have
$$K\left\{\frac{dN(t)}{dt}\right\} = -kK\{N(t)\} \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \ldots . \ldots . (22)$$

Now applying the property, Kamal transform of derivative of function, on (22), we have

$$\frac{1}{v} K\{N(t)\} - N(0) = -kK\{N(t)\} \dots \dots \dots \dots \dots \dots \dots (23)$$

Since at $t = 0, N = N_0 = 100$, so using this in (23), we have

$$\frac{1}{v} K\{N(t)\} - 100 = -kK\{N(t)\}$$

$$\Rightarrow \left(\frac{1}{v} + k\right) K\{N(t)\} = 100$$

$$\Rightarrow K\{N(t)\} = \frac{100v}{(1 + kv)} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (24)$$

Operating inverse Kamal transform on both sides of (24), we have

$$N(t) = 100 K^{-1}\left\{\frac{v}{(1 + kv)}\right\}$$

$$\Rightarrow N(t) = 100 e^{-kt} \dots \dots \dots, \dots \dots \dots \dots \dots \dots \dots \dots \dots (25)$$

Now at $t = 2$, the radioactive substance has lost 10 percent of its original mass 100 mg so $N = 100 - 10 = 90$, using this in (25), we have

$$90 = 100 e^{-2k}$$
$$\Rightarrow e^{-2k} = 0.90$$
$$\Rightarrow k = -\frac{1}{2} log_e 0.90 = 0.05268 \dots \dots \dots \dots \dots \dots \dots \dots (26)$$

We required t when $N = \frac{N_0}{2} = \frac{100}{2} = 50$ so from (25), we have

$$50 = 100 e^{-kt} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (27)$$

Putting the value of k from (26) in (27), we have

$$50 = 100 e^{-0.05268\, t}$$
$$\Rightarrow e^{-0.05268\, t} = 0.50$$
$$\Rightarrow t = -\frac{1}{0.05268} log_e 0.50$$
$$\Rightarrow t = 13.157 hours \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (28)$$

which is the required half-time of the radioactive substance.

XI.   CONCLUSION

In this paper, we have successfully developed the Kamal transform for solving the population growth and decay problems. The given applications show that the effectiveness of Kamal transform for solving population growth and decay problems. The proposed scheme can be applied for the continuous compound interest and heat conduction problems.

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